1.4 Euler Diagram Layout Techniques

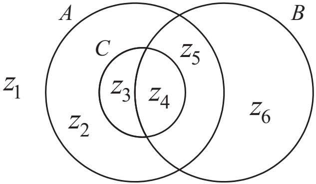
Euler Diagram Layout Techniques: Overview

- Dual graph based methods
- Inductive methods
- Drawing with circles

Including software demos.

How is the drawing problem stated?

Zones



$$
\begin{aligned}
z_1 &= \emptyset \\
z_2 &= A \\
z_3 &= AC \\
z_3 &= ABC \\
z_5 &= AB \\
z_6 &= B
\end{aligned}
$$

An abstract description is a set of zone descriptions (simply called abstract zones, or just zones).

Euler Diagram Layout Techniques: Generation Problem

Given an abstract description (a collection of zones), find an Euler diagram with that abstract description.

Euler Diagram Layout Techniques: Generation Problem

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Question

Can you draw this?

A, B, BC

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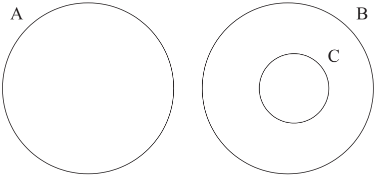
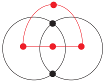
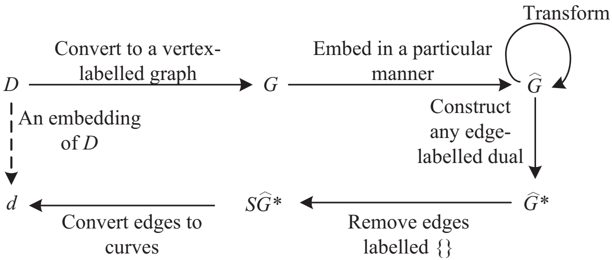
Solution





Dual Graph Generation

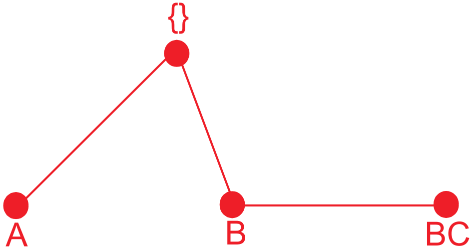


Dual Graph Generation

Problem

Draw a diagram with this abstraction using a dual graph method

A, B, BC



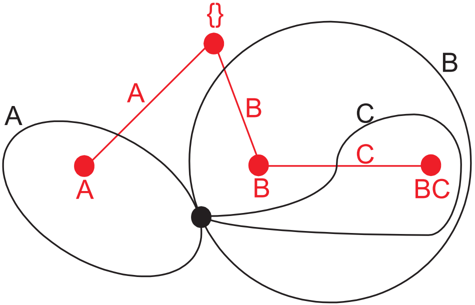
Step 1

Dual Graph Generation

Problem

Draw a diagram with this abstraction using a dual graph method

A, B, BC



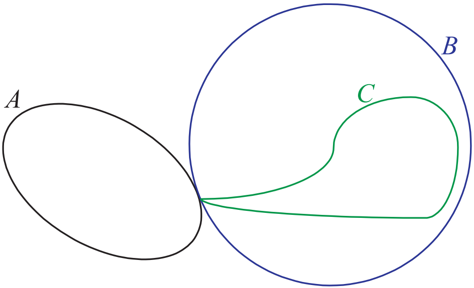
Step 2

Dual Graph Generation

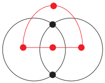
Problem

Draw a diagram with this abstraction using a dual graph method

A, B, BC



Step 3



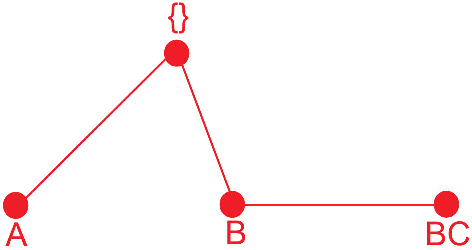
Dual Graph Generation

Problem

Draw a diagram with this abstraction using a dual graph method

A, B, BC

Step 1

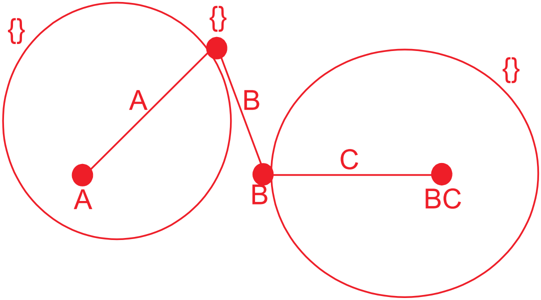


Dual Graph Generation

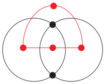
Problem

Draw a diagram with this abstraction using a dual graph method

A, B, BC



Step 1b

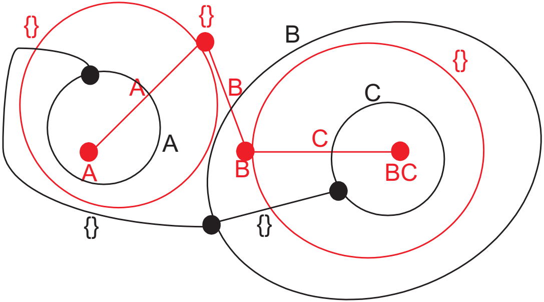


Dual Graph Generation

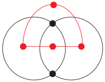
Problem

Draw a diagram with this abstraction using a dual graph method

A, B, BC



Step 2

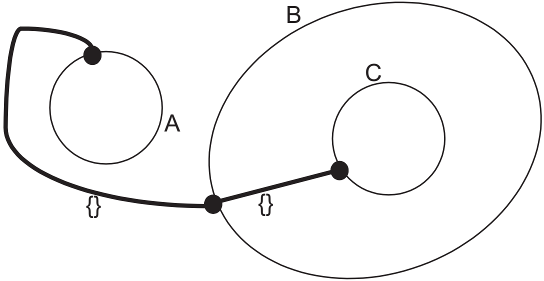


Dual Graph Generation

Problem

Draw a diagram with this abstraction using a dual graph method

A, B, BC



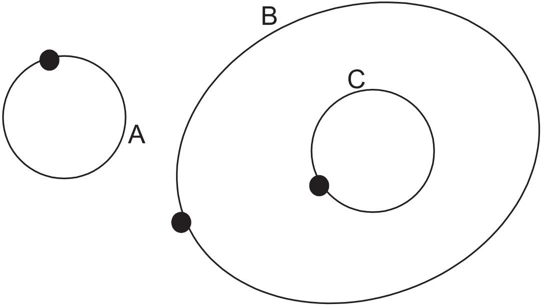
Step 2b

Dual Graph Generation

Problem

Draw a diagram with this abstraction using a dual graph method

A, B, BC



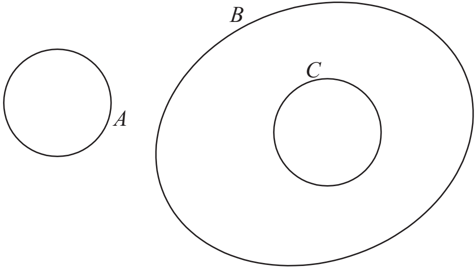
Step 2b

Dual Graph Generation

Problem

Draw a diagram with this abstraction using a dual graph method

A, B, BC

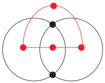


Step 3

Dual Graph Generation

Summary

Create dual graph (red)
Create dual of this graph
Remove edges labelled {} to create Euler graph
Convert Euler graph to Euler diagram



Dual Graph Generation

Advantages

- Can draw any abstract description
- Capable of enforcing different collections of properties

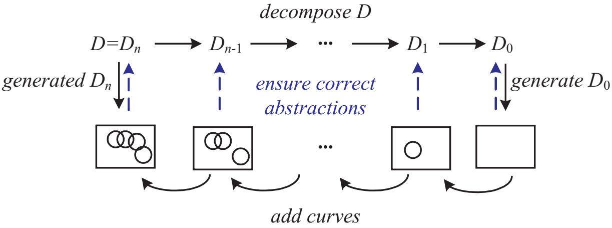
Disadvantages

- Computationally expensive
- Hard to identify duals that give rise to required properties
- Not all diagrams are aesthetically pleasing

Dual Graph Generation

DEMO

Inductive Generation

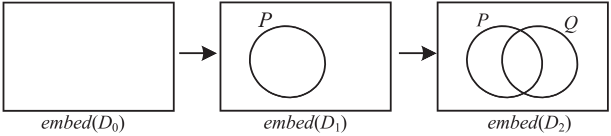


Inductive Generation

Problem

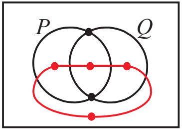
Draw a diagram with this abstraction using an inductive method

P, Q, PQ

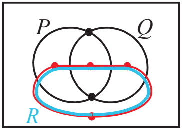


Inductive Generation

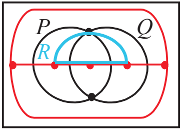
Adding curves using cycles



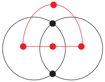
d_4



d_5



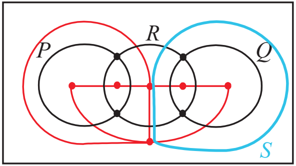
d_6



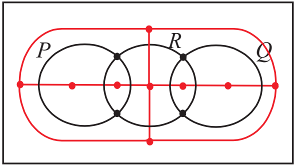
Inductive Generation

Problem

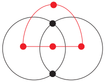
Adding curves using cycles: modifying the Euler dual



d_7



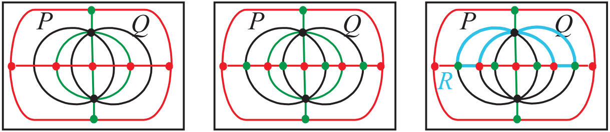
d_8



Inductive Generation

Problem

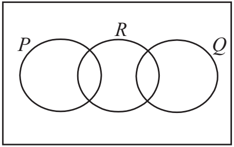
Adding curves using cycles: the hybrid graph



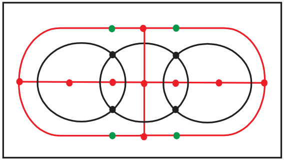
d_9 d_{10} d_{11}

Inductive Generation

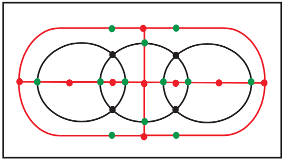
Creating the hybrid graph



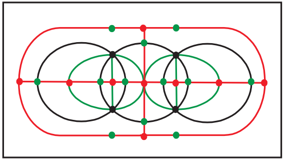
d_1



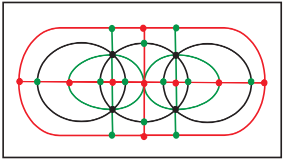
G_2



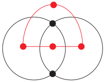
G_3



G_4



$HG(d_1)$

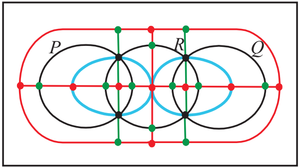


Inductive Generation

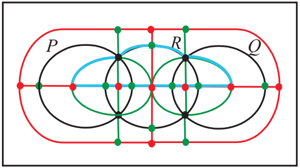
Add one curve at a time
Create hybrid graph
Find appropriate cycle to add desired curve
The properties of the cycle correspond to properties possessed by the resulting diagram

Inductive Generation

Example



d_{12}



d_{13}

Both diagrams have same abstraction, but only one possesses the simplicity property.

Inductive Generation

Advantages

Can draw any abstract description
Easily capable of enforcing different collections of properties:
easy to see from chosen cycle which properties are enforced

Disadvantages

Computationally expensive: can result in Hamiltonian cycles being sought
Not all diagrams are aesthetically pleasing

Inductive Generation

DEMO

Piercing Generation: Using Circles

This approach draws diagrams with circles

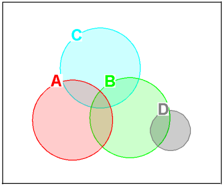
Advantages

- All diagrams drawn possess all of the six properties
- Diagrams are aesthetically pleasing
- Very efficient: polynomial time complexity

Disadvantages

- Cannot draw some abstract descriptions

Piercing Generation: Using Circles



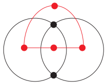
abstract syntax
A, B, AB, C, AC, BC, ABC, D, BD

Piercing Generation: Using Circles

Question

Is this drawable with circles?

A, B, AB, C, AC, BC, ABC, D, BD, CD, BCD

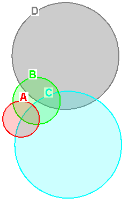


Piercing Generation: Using Circles

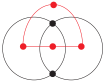
Question Is this drawable with circles?

A, B, AB, C, AC, BC, ABC, D, BD, CD, BCD

Answer Yes:



Question How can we identify whether an abstract description is drawable with circles?



Piercing Generation: Using Circles

Question

Is this drawable with circles?

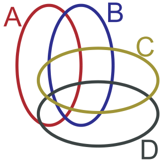
A, B, AB, C, AC, BC, ABC, D, AD, BD, CD, ABD, ACD, BCD, ABCD

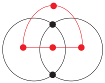
Piercing Generation: Using Circles

Question Is this drawable with circles?

A, B, AB, C, AC, BC, ABC, D, AD, BD, CD, ABD, ACD, BCD, ABCD

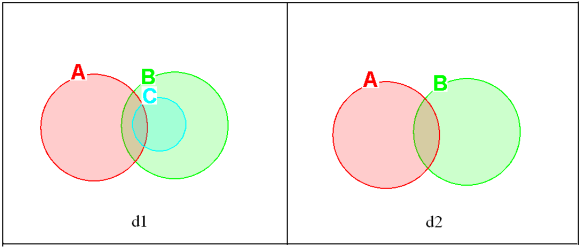
Answer No, but it can be drawn with ellipses:





Single Piercings

C contains two zones — Removing *C* yields:



- Zones combine when *C* is removed.
- *C* is a **single piercing** of *A*.
- At the abstract level:

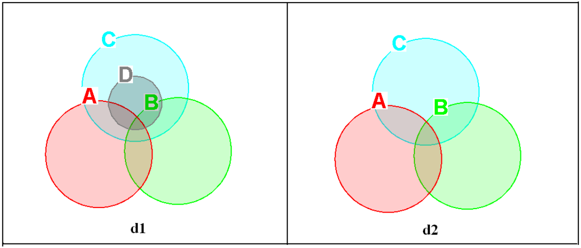
A, B, AB, **BC, ABC**
becomes
A, B, AB,

NOTE: BC and ABC differ only by A, the pierced curve

lose the two abstract zones containing *C*.

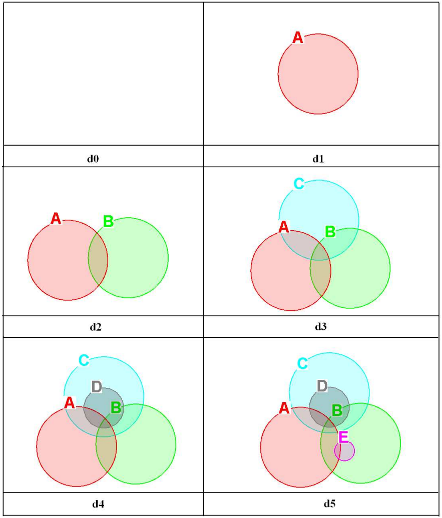
Double Piercings

D contains four zones. Removing *D* yields:

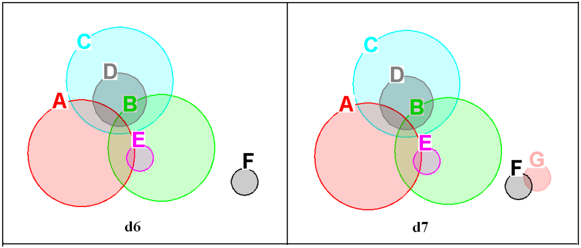


- Zones combine when *D* is removed.
- *D* is a **double piercing** of *A* and *B*.
- At the abstract level:
  A, B, AB, C, AC, BC, ABC, **CD, ACD, BCD, ABCD**
  becomes
  A, B, AB, C, AC, BC, ABC
  lose the four abstract zones containing *D*.

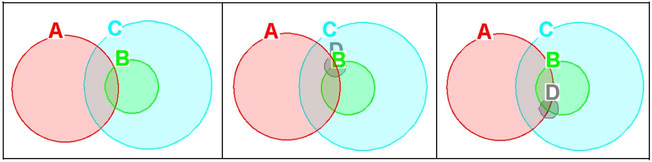
Building Diagrams from Piercings



Building Diagrams from Piercings: Disconnected Diagrams

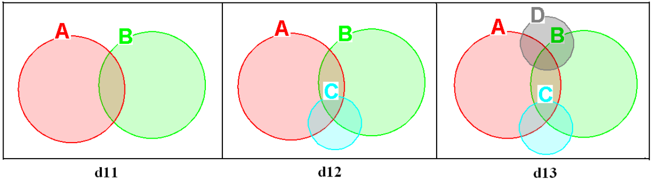


Building Diagrams from Piercings: Choices of embedding



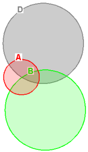
d8 d9 d10

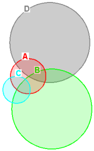
Building Diagrams from Piercings: Choices of embedding



Building Diagrams from Piercings: No choice of embedding

We want to draw
A, B, AB, C, AC, BC, ABC, CD, ACD, BCD, ABCD, E, AE, BE, ABE
[C, D and E are double piercings of A and B]



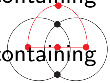


Cannot add E as a circle

Abstract Level: Inductive Definition

An abstract description, d, is **inductively pierced** if

1. d has no curve labels, or
2. d has a base piercing, λ_1 and d with the zones containing λ_1 removed is inductively pierced,
3. d has a single piercing, λ_1 and d with the zones containing λ_1 removed is inductively pierced, or
4. d has a double piercing, λ_1, of λ_2 and λ_3 such that d with the zones containing λ_1 removed is inductively pierced, and
   - no other curve label in d is outside-associated with λ_2 and λ_3 or
   - exactly one other curve label, λ_4, in d is outside-associated with λ_2 and λ_3 and
     - the curve labels containing λ_1 are the same as those containing λ_4 and λ_2, or
     - the curve labels containing λ_1 are the same as those containing λ_2 and λ_3.



Abstract Level: Inductive Definition

Theorem If d is an inductively pierced abstract description then d is drawable with circles, satisfying all wfc, in polynomial time.

Lemma If d is an inductively pierced abstract description with piercing curve label λ then $d - \lambda$ is inductively pierced.

Piercing Generation: Using Circles

DEMO